Game Theory: Some Preliminaries

Abbreviations

• Concepts and Learning Points
  • Battle of sexes
  • Best response function
  • Coin matching game
  • Free rider problem
  • Nash equilibrium
  • Non-zero-sum game
  • Nash equilibrium
  • Negative externality
  • Payoff matrix
  • Positive externality
  • Pure strategy
  • Random strategy
  • Strategy space
  • Zero-sum game

4.1 INTRODUCTION

Our economic theorization and analysis of terrorism and counter-terror measures will use some basic concepts and results of game theory. It will thus be useful to understand them beforehand. This chapter, not on terrorism per se, delves into game theory to the extent we’ll need it. Basic knowledge of intermediate microeconomics is all that is required.

In many real-world game situations, one person or team wins and the other loses, e.g., tennis, baseball, basketball, football, cricket and wrestling. As winning and losing cancel out in some
sense, such games are called **zero-sum games**. In regular warfare when one party or country wins and the other loses, one may be tempted to think that it is zero-sum game. But if we take into consideration the value of resources used or lost in war, it is a negative-sum, *not* a zero-sum, game. To see it suppose that party A wins and party B loses in a war. Let the values of winning and losing be $T$ and $-T$ respectively measured in dollars. War is costly in terms of resources, values of lives lost or injured, etc. Let these costs be $C_A$ and $C_B$ respectively. Thus the (net) payoffs to parties A and B are, respectively, $T - C_A$ and $-T - C_B$, and, both add up to $T - C_A - T - C_B = -(C_A + C_B) < 0$. In this sense it is a negative-sum game, and, more generally, a **non-zero-sum game**.

Most game situations in the context of economics and all game situations to be studied in this book are non-zero-sum games.

### 4.2 Defining a Game and the Central Question

A game is defined by three elements.

1. A set of players, say from 1 to $P$;
2. A set of strategies available to each player. We can call them $\{z_1\}, \{z_2\}, \ldots, \{z_P\}$;
3. An expression of payoff of each player which depends on strategies chosen by himself and other players.

It is crucial that payoffs of any particular player must depend not only on that player’s strategy or action, but also on some other player’s strategy. Otherwise, there is no strategic interaction or ‘game’ between that player and others.

In a two-person or two-party game, say between the U.S. (player 1) and N. Korea (player 2) hypothetically, we can think of the strategies for U.S. as $z_1 = \{\text{status quo, bomb, impose sanctions}\}$ and those for N. Korea as $z_2 = \{\text{status quo, bomb}\}$. Thus there are three strategies for the U.S. and two for N. Korea, altogether six strategy outcomes. Plus, we need to be given expressions or numbers representing payoffs to both U.S. and N. Korea associated with each outcome. Here is an example.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>N. Korea</td>
</tr>
<tr>
<td>status quo</td>
<td>status quo</td>
</tr>
<tr>
<td>status quo</td>
<td>bomb</td>
</tr>
<tr>
<td>bomb</td>
<td>status quo</td>
</tr>
<tr>
<td>bomb</td>
<td>bomb</td>
</tr>
<tr>
<td>sanctions</td>
<td>status quo</td>
</tr>
<tr>
<td>sanctions</td>
<td>bomb</td>
</tr>
</tbody>
</table>
4.3 Prisoner’s Dilemma

Table 4.1: Prisoner’s Dilemma Game

<table>
<thead>
<tr>
<th>Player A</th>
<th>Confess</th>
<th>Hang Tough</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>(-5,-5)</td>
<td>(-1,-10)</td>
</tr>
<tr>
<td>Hang Tough</td>
<td>(-10,-1)</td>
<td>(-2,-2)</td>
</tr>
</tbody>
</table>

Note that the same numbers for the U.S. and N. Korea may not mean the same magnitudes of payoffs, because their units may be different, e.g. US$ for the U.S. and “won” for N. Korea (its currency).

In the above example, strategies are discrete. Depending on the situation, they may be continuous. In a duopoly market with two firms, 1 and 2, for instance, their strategies may be the outputs they sell in a given period, $z_1$ for firm 1 and $z_2$ for firm 2, where outputs are measured on a continuous scale like wheat in tons or electricity in kilowatt. In this case, the strategy space of each player is infinite: 0 to $+\infty$.

In another dimension, the strategies could be deterministic (as in our preceding example and examples to come) or random. As an illustration of random strategies, consider a penalty shootout with two players, the striker and the goal-keeper. Each has three strategies with some probability distribution. For the goal-keeper they are: dive to the left with some probability, dive to the right with some probability or hold still with some probability. For the striker, it would be shoot left with some probability, shoot right with some probability and shoot straight with some probability. The probabilities must add to one for each player. When strategies are deterministic, they are called pure strategies and when they are probabilistic, they are called random strategies.

The central question that game theory seeks to answer is: which strategies will the players choose? Put differently, what is the solution concept that defines the equilibrium choice of strategies. The most widely used equilibrium concept is the Nash equilibrium, which will be introduced in the first example below. Altogether, we will analyze three game situations, which should suffice for our purpose in the subsequent chapters.

4.3 Prisoner’s Dilemma

It is a classic example introducing game theory. Imagine two prisoners, A and B, in custody who are suspects in a crime committed together. Both are interrogated separately and simultaneously without any contact with each other. Each has two choices or strategies: {confess, hang tough}. There are four strategy combinations. Table 4.4 summarizes the anticipated years in jail in different scenarios. The negative signs reflect that these are penalties, negative of payoffs. In each cell, the first number is for prisoner A and the second is for Prisoner B. If both confess, they will be sentenced five years each. If both hang tough, they will get two years. As an incentive for
confessing they are offered a deal that if one confesses and the other does not, the person who confesses get a much reduced sentence, one year in prison, whereas the person who does not gets a very stiff ten-year sentence. The $2 \times 2$ matrix is called the \textit{payoff matrix}.

Of course, if both prisoners deny and hang tough, they get the best deal jointly. But the problem is that they cannot communicate and hence there is no scope for them to cooperate. They are pretty much other own. A does not know for sure that B will hang tough and vice versa. The question is, which strategy will they rationally choose on their own? We need to use a reasonable equilibrium solution notion. The classic one is \textit{Nash equilibrium} or Nash-strategy, named after John Nash, a Nobel prize winner in economics.

Formally put, in a $P$-person game, strategies $\{z^N_1, z^N_2, \ldots, z^N_P\}$ would constitute Nash-equilibrium if for any player, say player $p$, $z^N_p$ is the best, i.e., the payoff maximizing, strategy, given the strategies by all other players, $\{z^N_1, \ldots, z^N_{p-1}, z^N_{p+1}, \ldots, z^N_P\}$. Differently put, Nash-equilibrium strategies are such that they are individually rational for each player in the game. The underlying notion of rationality is what makes the Nash equilibrium concept attractive particularly to economists.

In the Prisoner's dilemma, intuitively, a strategy combination (in our case strategies chosen by A and B) defines or constitutes Nash equilibrium if, given the strategy of all other players, no player has any incentive to deviate from his/her own strategy. Let us see whether the strategy combination $\{\text{confess, hang tough}\}$ for players A and B respectively that implies one-year and ten-year sentence to A and B respectively is a Nash equilibrium. One can easily see that it is not — because, given “confess” chosen by prisoner A, prisoner B would like to deviate from “hang tough” to “confess” which reduces his sentencing from ten to five years. By similar logic, $\{\text{hang rough, hang tough}\}$ and $\{\text{hang tough, confess}\}$ are not Nash equilibrium. In fact, $\{\text{confess, confess}\}$ is the only Nash equilibrium. Note the following.

\begin{itemize}
  \item[a.] Nash equilibrium is also called a non-cooperative equilibrium in the sense that the players decide their strategies on their own without coordinating or cooperating with others.
  \item[b.] Prisoner’s dilemma is a two-person, two-strategy game. In general, there can be game situations with more players. In the auto market, for example, there are many firms including General Motors, Ford, Toyota, Honda and Volkswagen.
  \item[c.] There can be more than two strategies. In the auto example, the number of product lines (models) can be a strategic variable and it can exceed two. Further, the number of different models would vary from one manufacturer to another.
  \item[d.] The number of strategies available for a player can be (technically speaking) infinite, e.g., annual production level of a commodity by a firm in a continuous scale, starting from zero upwards.
\end{itemize}
4.3. Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>(-1,1)</td>
</tr>
<tr>
<td>Tail</td>
<td>(-1,1)</td>
</tr>
</tbody>
</table>

Table 4.2: Coin Matching Game

In some game situations, there may not be any Nash equilibrium. The coin matching game, shown through Table 4.2, is an example, where both players have coins inside their palms and open their palms to show head or tail to each other simultaneously and if both coins match (show head or tail), player A wins and obtains $1 from player B and if the coins do not match, player B wins and gets $1 from player A. You can check that none of the four outcome satisfies the definition of Nash equilibrium. Notice that it is a zero-sum game.

Table 4.3: Battle of Sexes

<table>
<thead>
<tr>
<th>Patrice</th>
<th>Nathan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opera</td>
<td>(2,1)</td>
</tr>
<tr>
<td>Boxing Match</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

f. Similarly there are games, e.g., battle of sexes, where there are more than one Nash equilibrium. Nathan and Patrice like each other’s company very much. There are two options available to them on a Saturday: go to an opera or a boxing match. If one goes to opera and the other to the boxing match separately, each obtains zero utility. However, if they go to an event together, Patrice prefers opera to the boxing match, whereas Nathan prefers boxing match to opera. These utility payoffs are illustrated in Table 4.3. You can check that there two Nash equilibria in this ‘game’: {opera, opera} and {boxing match, boxing match}.

Nash equilibrium is a very useful concept for two reasons. First, it defines equilibrium in a non-cooperative situation which is natural and intuitive. Realize that since the two prisoners are being interrogated in separate cells and they have no means to communicate, coordinate or cooperate, they make choices on their own — which is individually rational. Secondly, we can compare Nash equilibrium to what the outcome will be in a cooperative environment. Notice in Table 4.4 that if the prisoners could cooperate and maximize their joint gains so to speak, they would both choose “Hang Tough,” i.e., deny the crime and their gains will be three less number years in prison (from five to two).
4.4 ‘Dormitory Game’: A Case of Negative Externality

Unlike the Prisoner’s dilemma where each player had two distinct (discrete) strategies to choose from, in this game as well as the next, players choose the level of their strategies, which is continuous. Imagine two adjacent rooms in a dormitory. Sandy stays in one and John is her neighbor, sharing a common wall which is poorly sound-proof. Both love to play music and at the same time of the day. Music volume or loudness is the choice variable or strategy of each ‘player’. Assuming that the volume can be raised continuously from zero to very loud, the number of strategies available to each player is infinite like points on a straight line, i.e., the strategy level is a continuous variable. Accordingly, the Nash equilibrium will be determined in a different way, although the concept of Nash equilibrium remains exactly the same: that is, combination of strategies, such that no single player has any incentive to deviate.

Let the volume chosen by Sandy and John be denoted by $v_s$ and $v_j$ respectively and let the utility functions from one’s own music be

$$U_s = U_s(v_s, v_j)$$
$$U_j = U_j(v_s, v_j).$$

(4.1)

The sign beneath each argument of the respective function indicates the nature of the cause-effect relation. The ‘±’ sign below $v_s$ in the $U_s(v_s, v_j)$ function means that, John’s music volume remaining the same, Sandy’s utility from the volume of her own music first increases and then falls. The ‘−’ sign below $v_j$ reflects that, for any given volume chosen by Sandy, an increase in volume set by John negatively affects Sandy’s utility. This is a case of negative externality.
4.4. A 'Dormitory Game': A Case of Negative Externality

**Figure 4.2: Best Response Functions and Nash Equilibrium in the Music Playing Game**

At different volumes played by John, Sandy’s utility functions are exhibited in Figure 4.1. It graphs three sound levels by John: \( v_j^1, v_j^2 \) and \( v_j^3 \) where \( v_j^1 < v_j^2 < v_j^3 \). For any given volume played by John, Sandy’s utility from the volume of her own music first increases and then falls. Notice the the higher the volume played by John, (a) the less is the utility for Sandy from the same volume she plays (reflecting negative externality) and (b) the higher is her own optimal volume (since her optimal volume is louder). Figure 4.1 implies that in order to counteract the negative externality, the louder John’s volume, the louder plays Sandy.

John’s utility function exhibits similar patterns. Further, in the above depiction we implicitly assume that the music volume set by either is not so loud that it is unbearable for the neighbor and playing music doesn’t make any sense for her/him.

Recall the definition of Nash equilibrium: it refers to those strategies of players such that no player benefits from deviating his/her respective strategy. In our context, Nash equilibrium translates to volume levels \( v_S^N \) and \( v_J^N \) if, given \( v_J^N \), \( v_S^N \) is the best strategy of Sandy, and, given \( v_S^N \), \( v_J^N \) is the best strategy of John.

Turning again to Figure 4.1, if John chooses volume \( v_j^1, v_j^2 \) or \( v_j^3 \), the best volume chosen by Sandy is \( v_S^1, v_S^2 \) and \( v_S^3 \) respectively. But in order to determine Nash equilibrium, we would need to assess Sandy’s best volume selection at each possible volume chosen by John as well as vice versa. Toward this end, let us graph each player’s best music volume level, given different volumes chosen by the neighbor. This is called the **best response function.** Generally speaking, the best response function in a two-person game is a graph of one player’s optimal strategy against different strategies chosen by the other. In this example, since each plays louder as the neighbor plays louder (see Figure 4.1 once again), their best response functions will be upward sloping. These are shown in panels (a) and (b) of Figure 4.2, where \( R_S \) is the best response function of Sandy and \( R_J \) is the same for John. It is normally the case that the best response function of the player whose strategy is measured along the horizontal axis will be steeper than that of the player.
Section 4.4: ‘Dormitory Game’: A Case of Negative Externality

whose strategy is measured along the vertical axis. Hence $R_S$ is drawn steeper than $R_J$.

The best response functions intersect at point $N$, shown in panel (c) of Figure 4.2, where the two strategy levels are $\{v^N_S, v^N_J\}$ for Sandy and John respectively. Realize that these strategies must constitute the Nash equilibrium, because, at volume $v^N_S$ chosen by Sandy, John’s best choice is $v^N_J$, i.e., he has no incentive to deviate from $v^N_S$, and, similarly, given $v^N_J$, Sandy has no incentive to deviate from $v^N_S$. The choices $\{v^N_S, v^N_J\}$ are consistent with each other in some sense and satisfy the definition of the Nash equilibrium.

It may be easier to understand Nash equilibrium through equations. Going one more time back to Figure 4.1, notice that the best strategy selection points of Sandy are such that the partial $\frac{\partial U}{\partial v_S} = 0$. The same will hold for John as well. Hence

$$\frac{\partial U_S}{\partial v_S}(v_S, v_J) = 0; \quad \frac{\partial U_J}{\partial v_J}(v_S, v_J) = 0.$$  

These are two equations in two variables, the simultaneous solutions of which must be the Nash solution — same as $v^N_S$ and $v^N_J$ in Figure 4.2. The important point to note is that when Sandy and John decide their music volumes independently or non-cooperatively, the negative externalities they cause on each other — reflected by the terms $\frac{\partial U_J}{\partial v_S}$ and $\frac{\partial U_S}{\partial v_J}$ — are not taken into consideration or internalized.

Two points may be noted. First, it is possible that the two best response functions do not meet; if so, there is no Nash equilibrium. Alternatively, they may meet at more than one point and in this case there are multiple equilibria. We ignore these possibilities since they are not of central interest to us. Second, negative externalities do not mean upward sloping best response function always. It depends on the nature of the game.

Like in the prisoner’s dilemma case, we now ask what music volumes Sandy and John will choose if they cooperatively decide. In general, any cooperative decision aims to maximize some measure of joint gains or utility to all. In our context it seems reasonable to suppose that if Sandy and John sit down cordially over coffee to decide the volumes they will choose, they will set the sum of their utilities, $U_S(v_S, v_J) + U_J(v_S, v_J)$, as the objective function, which they would wish to maximize by choosing $v_S$ and $v_J$.

The respective first-order conditions of maximizing this objective function will be the respective choice rules:

$$v_S: \frac{\partial U_S}{\partial v_S} + \left(\frac{\partial U_J}{\partial v_S}\right)_{\text{negative externality}} = 0$$  

$$v_J: \frac{\partial U_J}{\partial v_J} + \left(\frac{\partial U_S}{\partial v_J}\right)_{\text{negative externality}} = 0.$$  

These conditions ensure that any deviation from the chosen levels would not increase the respective utilities, i.e., they are consistent and satisfy the definition of the Nash equilibrium.
For simplicity of notation we have we have not written the arguments \((v_S, v_J)\) explicitly in the above equations. We must understand eqs. (4.3) and (4.4). At any given level of \(v_J\), the best possible level of \(v_S\) that maximizes the joint gains \(U_S(v_S, v_J) + U_J(v_S, v_J)\) must satisfy the marginal condition eq. (4.3). Similarly, eq. (4.4) is the condition that the cooperative solution of \(v_J\) must satisfy at a given level of \(v_S\). The cooperative solutions, say \(v_S^C\) and \(v_J^C\), are obtained by solving these two equations simultaneously.

The important point to note is that unlike the non-cooperative Nash equilibrium, the negative externality effects, namely, the negative effect of music volume set by Sandy on John (i.e. \(\partial U_J/\partial v_S\)) and that set by John on Sandy’s utility, (i.e. \(\partial U_J/\partial v_S\)), are included in eqs. (4.3) and (4.4), thus internalized in the cooperative decision making. The implication is that, at any given \(v_J\), the cooperative choice of \(v_S\) will be smaller, i.e., Sandy should choose a lower volume, than what she would if Sandy and John did not cooperate. Equivalently, the cooperative choice of \(v_S\) at different levels of \(v_J\) will lie to the left of the line \(R_S\) in panel (a) of Figure 4.2. This is shown by the red line in panel (a) of Figure 4.3 and marked \(R_S^C\). The same applies to the cooperative choice of \(v_J\) at different levels of \(v_S\), marked by \(R_J^C\) in panel (b) of Figure 4.3. We can call the red lines as **cooperative choice functions**. Compared to Nash behavior, they exhibit lower volume levels for each player given the volume choice of the other. In panel (c), the intersection of the cooperative choice functions defines the cooperative equilibrium. The cooperative solutions are \(v_S^C\) and \(v_J^C\).

The conclusion is that, compared to the non-cooperative Nash equilibrium, under cooperation each player would use less volume.
4.5 Firecracker Game: A Case of Positive Externality

Unlike the dormitory game, this an example where the action or strategy of one player exerts a positive effect on the welfare or payoff of other players. Imagine a neighborhood trying to raise funds voluntarily to buy fire crackers for July 4th celebration. If member A contributes more to the common fund, more fire crackers can be purchased which would bring more enjoyment to member B as well. Hence members would tend to free ride on another. Accordingly, the conclusion is the opposite of what was true for the dormitory game: Nash equilibrium will characterized by less provision of fire crackers than what is best for all members combined in a cooperative environment.

To see this clearly, suppose there are only two members, A and B. Let their voluntary contributions be $m_A$ and $m_B$ respectively, both continuous, ranging from zero to infinity. The common pool of funds is $m_A + m_B$, which translates into a volume of fire crackers that provide utility to both. Let

$$U_i = U_i(m_A + m_B), \quad U'_i(m_A + m_B) > 0,$$

where $i = A, B,$ be the respective utility functions. Consider A's utility for instance. Her contribution remaining the same, if B contributes more, the total contribution towards fire crackers will be higher. This enhances A's utility — and in this sense there is a positive externality from one player's strategy or action to another's utility.

Further, let $U''_i(m_A + m_B) < 0$. It means diminishing marginal utility — and that if, for example,
4.5. Firecracker Game: A Case of Positive Externality

When a person contributes some amount for fire crackers, there is a loss of utility from the contribution itself in terms of foregone consumption of other goods. Let the own utility loss associated with a contribution of $m_A$ by Member A be

$$\frac{1}{2} \cdot c_A \cdot m_A^2,$$

where $c_A > 0$ is a constant, so that the marginal utility cost of contribution is $c_A m_A$. Hence the higher the contribution the greater are its total and marginal utility cost. Similarly, let

$$\frac{1}{2} \cdot c_B \cdot m_B^2, \quad c_B > 0,$$

be the total utility cost for Member B. In general, $c_A \geq c_B$. The payoff or net utility of member $i$
4.5. Firecracker Game: A Case of Positive Externality

has the expression

\[ P_i \equiv U_i(m_A + m_B) - \frac{1}{2} \cdot c_i \cdot m_i^2. \]

Nash equilibrium refers to that pair of strategies, \( m_A^N \) and \( m_B^N \), such that A would maximize her payoff \( P_A \) with respect to \( m_A \) at \( m_A^N \), given that B chooses \( m_B^N \), and, likewise, B would maximize her payoff \( P_B \) at \( m_B = m_B^N \) given that A chooses \( m_A^N \) — so that there is no incentive to deviate from own strategy given the strategy of other player.

Refer to Figure 4.5, which depicts Member A's rational strategy when Member B chooses \( m_B^0 \). The upper quadrant depicts A's total utility from firecracker as a function of her own contribution \( m_A \) at Member B's contribution equal to \( m_B^0 \) and the total utility cost associated with her contribution. The difference between A's total utility from firecracker and her total utility cost of contribution is the curve \( P_A \). This is A's payoff and comparable to the utility functions in the previous example. Observe that when B contributes \( m_B^0 \), A maximizes her payoff by choosing her contribution equal to \( m_A^0 \). This is where the slope of her total utility curve, the marginal utility, is equal to the slope of her total cost line, equal to the marginal cost. The bottom quadrant illustrates A's rational choice in terms of marginal utility and marginal cost.

Algebraically, the individually rational choices are governed by the respective first-order
Figure 4.7: Best Response Functions and Nash Equilibrium in the Firecracker Contribution Game

![Figure 4.7: Best Response Functions and Nash Equilibrium in the Firecracker Contribution Game](image)

conditions of maximizing the payoff $P_i$ with respect to $m_i$. These are:

As choice rule for $m_A$:
$$\frac{\partial U_A}{\partial m_A} = U'_A(m_A + m_B) = c_A m_A$$

B's choice rule for $m_B$:
$$\frac{\partial U_B}{\partial m_B} = U'_B(m_A + m_B) = c_B m_B,$$

the solutions of which constitute the Nash equilibrium.

As in the previous example, it will be instructive to derive the best response functions and see the Nash equilibrium graphically. We begin with the rational/optimal choice (contribution) of Member A. Panel (a) of Figure 4.6, a continuation of the lower quadrant of Figure 4.5, depicts optimal choice of $m_A$ at two levels of $m_B$: $m_B^0$ and $m_B^1$ where $m_B^1 > m_B^0$. We see that $m_A^1 < m_A^0$. Recall that at an increase in $m_B$ implies an inward shift of the marginal utility curve of Member A. Hence the marginal utility curve of A at $m_B = m_B^1$ lies to the left of that at $m_B = m_B^0$. As a result, the optimal contribution by A less when $m_B = m_B^1$ than when $m_B = m_B^0$.

The best response function of A, $R_A$, is thus downward sloping, shown in panel (a) of Figure 4.7. Similarly, the best response function of B, $R_B$, is downward sloping also; see panel (b).\(^1\)

The downward slope of the best response functions follows from the positive externality of one member’s contribution toward the utility of the other, as a result of which if one member contributes more, the other tends to free ride. As we learned in the previous example, Nash equilibrium is where the two best response functions intersect: at point $N$ in panel (c).

The central qualitative feature of the Nash non-cooperative equilibrium in this example is that there will be an under-provision of contributions since each member has an incentive to shirk and free ride. We can see this by analyzing the cooperative solution and comparing it with the

\(^1\)For simplicity, the $R_A$ and $R_B$ lines are shown as straight lines, but they need not be.
non-cooperative Nash solution. As before, we suppose that when members cooperate and jointly
decide the individual contributions, the objective is to maximize the sum of payoffs to all members.
In the present context of two members, it is equal to

\[ \mathcal{P}(m_A + m_B) \equiv P_A + P_B = U_A(m_A + m_B) - \frac{1}{2} c_A m_A + U_B(m_A + m_B) - \frac{1}{2} c_B m_B. \]  

(4.7)

Notice how the marginal impact of \( m_A \) or \( m_B \) on collective payoff under cooperation differs from
that on own welfare. It is the beneficial effect of \( m_A \) (or \( m_B \)) on Member B’s (Member A's) welfare
which is now “internalized” in that such positive externalities will be taken into consideration under
cooperative behavior. This was not the case when members choose their strategies independently
keeping in mind their own welfare or payoff only. The following first-order (marginal) conditions
govern the cooperative choice of strategies:

Cooperative choice of \( m_A \):

\[ \frac{\partial U_A}{\partial m_A} + \frac{\partial U_B}{\partial m_A} = U'_A(m_A + m_B) + U'_B(m_A + m_B) = c_A m_A \]  

(4.8)

Cooperative choice of \( m_B \):

\[ \frac{\partial U_B}{\partial m_B} + \frac{\partial U_A}{\partial m_B} = U'_B(m_A + m_B) + U'_A(m_A + m_B) = c_B m_B. \]  

(4.9)

**Figure 4.8: Nash Non-Cooperative Versus Cooperative Equilibria in the Fire Cracker Contribution Game**

The solutions of these equations, say \( m^C_A \) and \( m^C_B \), are the cooperative solutions of strategies.
These are illustrated and compared with the non-cooperative solutions in Figure 4.8. Since the
positive externality of one member’s contribution on the other’s utility is internalized, given
the contribution of one member, the cooperative choice of contribution by the other must be
higher. Accordingly, the cooperative choice functions $R_A^c$ and $R_B^c$ lie to the right or above the best response functions under non-cooperation. This is exactly the opposite of what was true for the dormitory game. The cooperative equilibrium is indicated at point C where the two cooperative choice functions intersect. Depending on how far apart the cooperative choice and best response functions are compared to the non-cooperative equilibrium, cooperative contributions by both members may be higher or that by one member is higher while that by the other is less. If only the members total utility and total cost functions are very different from each other, possibilities (b) or (c) will occur; otherwise it will be (a), where both members are urged to contribute more. Regardless of whether one member or both would be contributing more, it can be shown that the total contribution will always greater under cooperation than under non-cooperation.

The general conclusion is that, compared to the non-cooperative equilibrium, in the cooperative equilibrium, at least one member will contribute more and total contributions will be greater.

4.6 **Other Forms of Games**

For example, sequential game. *To be written.*
Exercises

4.1 Consider the ‘discreet’ version of the dormitory game. Suppose Sandy and John can choose two levels of sound ‘loud’ (L) and ‘soft’ (S).

<table>
<thead>
<tr>
<th></th>
<th>John</th>
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<tbody>
<tr>
<td>Sandy</td>
<td></td>
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<tr>
<td></td>
<td>S</td>
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<tr>
<td>S</td>
<td>(6,3)</td>
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<tr>
<td>L</td>
<td>(5,4)</td>
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4.2 Various countries attempt to attract domestic and foreign investment by offering financial concessions in the forms of different investment incentives. In 2002, it is estimated that Vietnam provided incentives worth 0.7% of its GDP; in 1996, U.S. state and local governments nearly US$47 billion to attract investments. In other words, states compete with each other and countries compete with each other to attract investment. Let $s_A$ and $s_B$ denote the respective subsidies. Define the benefit functions from subsidies be defined as

$$B_A(s_A, s_B); \quad B_A(s_A, s_B).$$

The signs of marginal effects are based on the following. Businesses are looking for a host state or country to locate their production facilities. States or countries are vying with each other to attract businesses. Own subsidies would tend to increase investment inflow, leading to job creation and income in the future, whereas subsidies by other states or countries would tend to attract away investment and therefore negatively the future economic prospects.